Lossless Tapers, Gaussian Beams, Free-Space Modes: Standing Waves Versus Through-Flowing Waves

Antonio-D. Capobianco, Michele Midrio, Carlo G. Someda and Stefano Curtarolo*

Istituto Nazionale per la Fisica della Materia

Dipartimento di Elettronica e Informatica

Università degli Studi di Padova

Via G. Gradenigo 6/A 35131 Padova, Italy.

Abstract

It was noticed in the past that, to avoid physical inconsistencies, in Marcatili's lossless tapers through-flowing waves must be drastically different from standing waves. First, we reconfirm this by means of numerical results based on an extended BPM algorithm. Next, we show that this apparently surprising behavior is a straightforward fallout of Maxwell's equations. Very similar remarks apply to Gaussian beams in a homogeneous medium. As a consequence, Gaussian beams are shown to carry reactive powers, and their active power distributions depart slightly from their standard pictures. Similar conclusions hold for free-space modes expressed in terms of Bessel functions.

*Present address: Dipartimento di Fisica G. Galilei, Universitá degli Studi di Padova.

1 Introduction.

In 1985, Marcatili infringed a historical taboo, showing that one can conceive and design, at least on paper, dielectric tapers and bent waveguides that are strictly lossless [1]. The key feature, shared by the infinitely many structures which obey Marcatili's recipe, is that the phase fronts of their guided modes are closed surfaces. Phase fronts which extend to infinity in a direction orthogonal to that of propagation entail radiation loss: closed fronts can overcome this problem. Shortly later, however, it was pointed out that this recipe could generate some incosistencies [2]. In fact, a traveling wave with a closed phase front is either exploding from a point, or a line, or a localized surface, or collapsing onto such a set. In a lossless medium and in the absence of sources or sinks, this picture is untenable. On the other hand, it was also pointed out in ref.[2] that a standing wave with closed constant-amplitude surfaces is perfectly meaningful under a physical viewpoint. Therefore, a through-flowing wave through any of Marcatili's lossless tapers or bends has to be described in this way: the incoming wave must be decomposed as the sum of two standing waves, of opposite parity with respect to a suitable symmetry plane (or, more generally, surface). The output wave is then found as the sum of the values taken by the two standing waves at the other extremity of the device. Another point raised in ref.[2] was that very similar remarks apply to Gaussian beams in free space. If applied literally to a traveling wave, the standard mathematics which is found in any textbook on Gaussian beams would entail that such beams either explode from their waist plane or implode from it: once again, a physically meaningless picture.

Later on, the literature showed that these problems were not dealt with for a long time. Recently, though, we observed several symptoms of a renewed interest in low-loss [3, 4, 5]

and lossless [6, 7] tapers or bends. This induced us to try to go beyond the results of ref.[2], aiming at clarifying more deeply the difference between through-flowing and standing waves in Marcatili's tapers and in Gaussian beams. Our new results, reported in this paper, can be summarized as follows. In Section 2, we show that a numerical analysis (based on an extended BPM algorithm) of Marcatili's tapers reconfirms that indeed through-flowing waves are drastically different from standing waves. The latter ones match very well the analytical predictions of the original recipe given in ref.[1], but through-flowing waves have open wave fronts, so that they do not entail any physical paradox. In Section 3, we provide an analytical discussion of why, in contrast to classical cases like plane waves in a homogeneous medium or guided modes in longitudinally invariant waveguides, through-flowing waves in Marcatili's tapers are so different from standing waves. We show that the difference is a straightforward fallout of Maxwell's equations. Although this entails that through-flowing waves in Marcatili's tapers are never strictly lossless, nonetheless our numerical results reconfirm that the recipes given in ref.[1] do yield extremely low radiation losses. In Section 4 we address the very similar problem of Gaussian beams in a homogeneous medium. We show that physical inconsistencies affecting the naive picture of a traveling Gaussian beam disappear, as soon as Maxwell's equations are handled with sufficient care. In Section 5, we focus our attention on those waves which were identified in ref.[2] as the true free-space modes (as opposed to Gaussian beams). Also for these we show that the picture of a through-flowing wave is affected in some of its significant features, if Maxwell's equtions are used to go a few steps further than what had been done in ref.[2].

The common key feature shared by Marcatili's tapers, Gaussian beams, and free-space modes, is that in general the Poynting vector is not simply proportional to the square of the

modulus of the electric field. The actual power distribution in space is more complicated. In particular, in contrast to classical cases, through-flowing fields in these problems are always characterized by nonvanishing reactive powers, which reach their highest levels in the proximity of the waist of the taper or beam. This indicates that through-flowing waves do not have all the features of a *pure* traveling wave, which, by definition, has a standing-wave ratio identical to 1, and thus cannot carry any reactive power.

2 Marcatili's tapers: numerical results.

The geometry of Marcatili's tapers can possibly be very complicated (e.g., see ref.[8]). Here, however, we prefer to adopt a simple shape, to avoid that geometrical features may blur the basic physics we were trying to clarify. The results on which we focus in this Section refer to a single-mode taper whose graded-index core region is delimited by the two branches of a hyperbola (labels A and A' in Figs.(1) and (2)), and has a mirror symmetry with respect to its waist plane. According to the terminology of ref.[1], this is a "superlinear" taper, with an index distribution

$$n = \begin{cases} n_0\sqrt{1 + 2\Delta/(\cosh^2\eta - \sin^2\theta)} & \text{for } \theta_1 < \theta < \theta_2 \\ n_0 & \text{for } \theta_1 > \theta > \theta_2 \end{cases} \tag{1}$$

where η and ϑ are the elliptical coordinates, in the plane of Figs.(1) and (2). Fig.(1) refers to standing waves, of even (part a) and odd (part b) symmetry with respect to the waist plane. The closed lines are constant-amplitude contour plots. They are essentially elliptical, so they agree very well with the predictions of ref.[1].

As mentioned briefly in the Introduction, these results were generated using an extended BPM, which deserves a short description. It is well known that standard BPM codes are suitable to track only traveling waves, as they do not account for backward waves. Our code (which uses a Pade's operator of order (5,5)) generates a traveling wave, but the direction of propagation is reversed whenever the wave reaches one of the taper ends. In order to generate a single-mode standing wave, each reflection should take place on a surface whose shape matches exactly that of the wave front. This is very difficult to implement numerically, especially as long as the wave front shape is the unknownfeature one is looking for. But the problem can be circumvented, by letting each reflection take place on a *phase-conjugation* flat mirror. Our code adopts this solution, and calculates then, at each point in the taper, the sum of the forward and backward fields. The process stops when the difference between two consecutive iterations is below a given threshold.

Fig.(2) refers to a through-flowing wave. The almost vertical dark lines in part a) are the phase fronts. They are drastically different from those predicted by the analytical theory of ref.[1], which are exemplified in the same figure as a set of confocal ellipses. Note that the through-flowing wave has been fabricated numerically in two ways. One was simply to launch a suitable transverse field distribution, and track it down the taper, with a standard BPM code. The other one was to calculate the linear combination (with coefficients 1 and j) of the even and odd standing waves shown in Fig.(1). The results obtained in these two ways are undistinguishable one from the other. This, altogether, proves that indeed through-flowing waves are drastically different from standing ones. In particular, as we said in the Introduction, through-flowing waves are totally free from any untenable feature under the viewpoint of energy conservation.

Fig.(2.b) shows a field amplitude contour plot for the same through-flowing wave as in Fig.(2.a). It indicates that, in spite of all the matters of principle which make a through-flowing wave different from a standing one, its propagation through the taper is indeed almost adiabatic. Therefore, as anticipated in the Introduction, insertion losses of Marcatili's tapers are very low, at least as long as the length to width ratio is not too small, although not strictly zero. A typical example is shown in Fig.(3). It refers to a taper like that of Figs.(1) and (2), whose total length is $2.5\mu m$, whose waist width is $0.55\mu m$, and whose initial (and final) width is $1.65\mu m$. The BPM calculations yield a lost power fraction of 1.4×10^{-4} at a wavelength of $1.55\mu m$.

3 Marcatili's tapers: analytical remarks.

For the sake of clarity, let us restrict ourselves to the case of two-dimensional tapers, like those of the previous section, where the geometry and the index distribution are independent of the z coordinate, orthogonal to the plane of the figures. However, our remarks will apply to 3-D structures also.

The index distributions that were identified in ref.[1] are such that the TE modes (electric field parallel to z) satisfy *rigorously* a wave equation which can be solved by separation of variables. Obviously, the same equation is satisfied rigorously by the transverse component of the magnetic field, as well. However, in general, if we take two identical solutions of these two wave equations (except for a proportionality constant), it is easy to verify that they do not satisfy Maxwell's equations. This statement could be tested, for example, on the superlinear taper of the previous Section. However, this proof would be mathematically cumbersome,

requiring use of Mathieu functions of the fourth kind, which satisfy the wave equation in the elliptic coordinate system. A much simpler, yet enlightening example, is the device which was referred to in ref.[1] as "linear taper" : a wedged-shape region, with a suitable index distribution, where only one guided mode can propagate in the radial direction. It is perfectly legitimate to say that the dependence of $\mathbf{E}_z$ on the radial coordinate is expressed by a Hankel function, whose imaginary order, $i\nu$, is related to the features of the individual taper [1]. What one cannot extrapolate from this, is that the same holds for the magnetic field. In fact, let us calculate the curl of the electric field. We find that the azimuthal component of the magnetic field is proportional to the *first derivative* of the Hankel function with respect to its argument (proportional to the radial coordinate). This derivative is never proportional to the function itself. This is a drastic difference with respect to plane waves, or to guided modes of longitudinally invariant waveguides, where the derivative of an exponential function (expressing the dependency on the longitudinal coordinate) remains proportional to the function itself. We see that in Marcatili's tapers, although $\mathbf{E}_z$ and $\mathbf{H}_\vartheta$, on any wavefront, have identical dependencies on the transverse coordinates, nevertheless it is troublesome to define a wave impedance, because they do not vary in identical fashions along the coordinate of propagation. It is equally risky to derive claims [1] regarding the Poynting vector from just the spatial distribution of the electric field, skipping the details of the magnetic field.

Let us strengthen our point with a few calculations, which aim at proving explicitly that a TE wave, whose radial dependence is expressed by a Hankel function of imaginary order, $H_{i\nu}$, cannot be a pure *traveling* wave along a linear taper. As we just said, if $\mathbf{E}_z$ is proportional to $H_{i\nu}$, then Maxwell's equations say that $\mathbf{H}_\vartheta$ is proportional to $iH'_{i\nu}$. The

radial component of the Poynting vector is proportional to $iH_{i\nu}(H'_{i\nu})^*$. In a purely traveling wave, by definition there is no reactive power flowing in the direction of propagation. In the case at hand, this would imply $|H_{i\nu}|^2 = constant$ along the radial direction, a requirement which cannot be satisfied by Hankel functions. Incidentally, note that exponential functions, which describe traveling plane waves and modes of longitudinally invariant waveguides, do satisfy this type of requirement. Coming back to the linear taper, it is easy to show that the requirement which chararcterizes a purely standing wave - zero active power in the radial direction - is satisfied by Bessel functions of order $i\nu$. This reconfirms that the exact modes of the lossless tapers found in ref.[1] are only standing waves. *Any* through-flowing wave along such a taper must be expressed as a linear combination of such standing waves. This point had already been stated in ref.[2]; the new contribution of this paper consists in showing that the impossibility of expressing a traveling wave in terms of one simple function of the coordinate along which it propagates (say, the Hankel function in the previous example) is a consequence not of singularities, but of the inherent nature of partially standing wave which characterizes such functions.

4 Analysis of Gaussian beams.

One of the points raised in ref.[2] was that the usual picture of a Gaussian beam may run into the same physical incosistency as a single traveling wave in a Marcatili taper. In fact, if we read the mathematics of Gaussian beams in a literal way, we find that the phase fronts are ellipsoidal surfaces, which are closed surfaces. A traveling wave with a closed phase front is either exploding from a localized source, or imploding onto a localized absorber. In a lossless

homogeneous medium, without sources, this is untenable. For a loosely focused beam, whose phase fronts are almost flat, the field amplitude is negligibly small in the regions - far from the beam axis - where the two “halves” of a phase front (one half on each side of the beam waist) meet. Consequently, the point that the phase fronts are closed surfaces appears not to be, in such a case, of practical relevance. On the contrary, in a tightly focused beam this fact is not irrelevant, and could explain some discrepancies between experiments and the simplest theories, which have been observed and reported in the literature. Once again, as noted in ref.[2], a standing wave with closed constant-amplitude surfaces is physically meaningful, regardless of how tightly it is focused. Henceforth, a traveling Gaussian beam which passes through its waist plane can be modeled correctly as the sum of two standing waves, of opposite parity with respect to the beam waist plane.

In this Section, we will show that the difference between traveling and standing waves - a deep difference as a matter of principle - can be explained as in the previous Section, using Maxwell's equations. Furthermore, this procedure will enable us to find quantitative criteria to assess when these changes with respect to the naive theory (where the Poynting vector is simply taken to be proportional to the square of the electric field) become of practical relevance. We will also show that the signs of the terms which are usually neglected in the power distribution depend on the beam polarization. To underline this, we speak of transverse-electric (TE) and transverse-magnetic (TM) Gaussian beams, in contrast to the classical $TEM_{m,n}$ terminology. We will deal explicitly only with the TE case. The reader may easily derive the TM case by duality.

In the paraxial approximation around the z-axis, $\left|\frac{\partial^2 \phi}{\partial z^2}\right| \ll 2k \left|\frac{\partial \phi}{\partial z}\right|$, the scalar Helmholtz

equation, in an indefinitely extended homogeneous medium, becomes

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2jk \frac{\partial \phi}{\partial z} \tag{2}$$

where $\mathbf{E} = \phi(x, y, z) \exp(-jkz)$. The features which we want to outline can be extracted from any solution of (2). So, let us focus on the simplest one, the so-called TEM_{00} mode, namely

$$\mathbf{E} = \mathbf{E}_0 e^{-jkz} e^{-jP} e^{-j\frac{k}{2q}r^2} \tag{3}$$

where

$$\frac{1}{q(z)} = \frac{1}{R(z)} - 2j\frac{1}{kw^2(z)} \quad , \quad P(z) = -j \ln\left[1 + \frac{z}{q_0}\right] \tag{4}$$

R being the radius of curvature of the beam phase front, and w the beam width at the 1/e amplitude level. Their z-dependence can be found in any textbook (e.g., ref.[9], Sect. 3.2).

It is elementary, starting from Maxwell's equations, to show that in the paraxial approximation *all* the components of the electric field vector, *and all* those of the magnetic field vector, satisfy (2). What is usually taken for granted is that, for a beam whose transverse electric field, say $\mathbf{E}_x$, is expressed by (3), the transverse magnetic field $\mathbf{H}_y$ is also of the form (3), so that the wave impedance $\mathbf{E}_x/\mathbf{H}_y$ is constant in space.In reality, this is not true: if we insert $\mathbf{E}_x$ expressed by (3) into Maxwell's equation $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$, we get

$$\mathbf{H}_y = \frac{\mathbf{E}_x}{-j\omega\mu} S \tag{5}$$

where

$$S = -jk + 2j\frac{1}{kw^2} - jk\left[\frac{2r^2}{k^2w^4} - \frac{r^2}{2R^2}\right] + \left[\frac{2r^2}{Rw^2} - \frac{1}{R}\right] \tag{6}$$

For $\mathbf{H}_y$ to be proportional to $\mathbf{E}_x$ and the wave impedance to be equal to $\eta = \sqrt{\mu/\epsilon}$, this expression should reduce to its first term. The second term, small and independent of the transverse coordinates, is insignificant. The following terms are small compared to k, but not negligible, not even in the paraxial approximation, as the reader can check by calculating the second derivative with respect to z and comparing it to (6) multiplied by k.

What can make these terms important (at least as a matter of principle) is that they depend on the transverse coordinates. The third term in (6) affects the real part of the Poynting vector, so that the active power density flowing through any cross section of the beam is not simply $|\mathbf{E}_x|^2/(2\eta)$. The last term gives rise to an imaginary component of the Poynting vector, indicating that there is reactive power present in the beam - a feature in contrast with the naive model of the beam as a pure traveling wave. Let us look at both terms in more detail.

The third term is the sum of two quantities of opposite sign, with identical dependence on the transverse coordinates through $r^2 = x^2 + y^2$, but different dependences on z through $1/R$ and $1/w^2$. Their sum cancels out exactly for $z = \pm\pi w_0^2/\lambda = z_R$, i.e. at the two extremes of the so-called Rayleigh range (w_0 being the spot size at the beam waist). On those two planes, but only there, the wave impedance is independent of r, and equal to η. Within the Rayleigh range, the positive contribution dominates, and the wave impedance (equal to η only for $r = 0$) decreases as the distance from the z-axis grows. For example, on the waist plane, where $1/R = 0$, at $r = 0.3z_R$ (where the field is still appreciable, in a very tightly

focused beam) the wave impedance is about 5% smaller than η. On the contrary, out of the Rayleigh range the sum in question becomes negative. Its magnitude reaches a maximum at a distance $z = \pm\sqrt{3}z_R$, and then decays to zero as z tends to infinity. The corresponding corrections on the wave impedance are negligible where the field amplitude is significant. Therefore, we may conclude that the usual TEM model is perfectly adequate out of the Rayleigh range.

The last term in (6), giving rise to reactive power, also consists of two contributions of opposite sign, whose sum cancels out along the hyperboloid $r^2 = w^2/2$ and has odd parity with respect to the waist plane, $z = 0$. Poynting's theorem, in a lossless medium and with no sources, leads then to the conclusion that electric and magnetic energy densities are not equally stored, at all points, in a Gaussian beam. In the TE case, there is more electric than magnetic energy stored near the beam axis, and this becomes more evident as one gets closer to the waist. The reverse is true in the periphery. Note that, with a simple change of variable $u = 2r^2/w^2$, and integrating by parts, it is easy to verify that for any z

$$\iint_{-\infty}^{+\infty} \frac{|\mathbf{E}|^2}{R} \left(1 - 2\frac{r^2}{w^2}\right) dxdy = 0 \tag{7}$$

The net flux of reactive power through any plane orthogonal to the beam axis is zero. Therefore, any "space slice" between such planes is resonant, i.e. stores equal amounts of magnetic and electric energy. But *locally*, this is not the case. Also note that these findings match the previously outlined points on active power. Indeed, if on the waist plane the wave impedance is smaller in the periphery compared to the center, then the ratio of magnetic to electric energy has to be larger in the periphery - and in fact it is. Finally,

the interested reader can calculate, through Maxwell's equations, the $\mathbf{H}_z$ component of the magnetic field, and then the corresponding imaginary y-component of the Poynting vector. Its sign reconfirms the previous statement: electric energy stored around the axis is more than the magnetic one, the opposite is true in the periphery.

5 Analysis of free-space modes.

One of the key issues of ref.[2] was to show that a through-flowing beam in free space can be correctly modeled, without running into inconsistencies or paradoxes, as the superposition of two standing waves of opposite parities. The typical example discussed in detail in ref.[2] was a wave whose electric field, parallel to the z-axis, is expressed, in cylindrical cordinates r, ϑ, z (see Fig. 4), as

$$\mathbf{E}_z(r, \vartheta) = J_0(kr) + jJ_1(kr)\sin(\vartheta) \tag{8}$$

where J_0, J_1 are Bessel functions of the first kind of orders $0, 1$, respectively, and $k = 2\pi/\lambda$ is the free-space wave number. In ref.[2], no explicit statements were made on how the power of this wave is distributed in space. However, it is legitimate to infer from the silence on this point, that it was taken for granted in ref.[2] what had been stated in ref.[1], namely that the Poynting vector was everywhere proportional to the square of the modulus of the electric field, since free space is a homogeneous medium. In the previous Sections, we have shown that this attitude is not justified when dealing with Marcatili's tapers or with Gaussian beams. In this Section we will prove that it is also erroneous in the case of free-space modes.

Let us first calculate the Poynting vector as $|\mathbf{E}|^2/2\eta_0$, where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the free-space

impedance. It is straightforward to find that it has just a radial component expressed by

$$\mathbf{P}_r = \frac{k}{2\omega\mu_0}\left[J_0^2(kr) + J_1^2(kr)\sin^2(\vartheta)\right] \tag{9}$$

This expression does not match with the idea of a through-flowing beam. As a matter of fact, the flow lines for $\mathbf{P}_r$, depicted in Fig. (5.a), clearly give the feeling of an exploding wave, rather than of a through-flowing beam.

Let us now see what we find when we proceed rigorously, in the same way as in the previous Sections. We calculate the magnetic field (two components) as the curl of eq.(8) in cylindrical coordinates, then the Poynting vector (radial and azimuthal component). We take its real part, and express it in terms of its cartesian components, in the reference frame shown in Fig.(4). These calculations yield:

$$\Re\{\mathbf{P}_x\} = -\frac{k}{2\omega\mu_0}\left[J_0^2(kr) + J_1^2(kr) - 2J_0(kr)\frac{J_1(kr)}{r}\sin(2\vartheta)\right] \tag{10}$$

$$\Re\{\mathbf{P}_y\} = -\frac{k}{\omega\mu_0}\left[\left(J_0^2(kr) + J_1^2(kr)\right)\sin^2(\vartheta) + 2J_0(kr)\frac{J_1(kr)}{r}\cos(2\vartheta)\right] \tag{11}$$

These results describe correctly a flow of active power, essentially in the direction of the y-axis. For example, to stress the fundamental difference with respect to (9), the fact that the sign of $\Re(\mathbf{P}_y)$ remains negative for $\vartheta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ is perfectly adequate to describe a flow from the $y > 0$ half space towards the $y < 0$ half space.

This is confirmed by Fig(5.b), where the flow lines for $\Re\{\mathbf{P}\} = \Re\{\mathbf{P}_x\}\hat{\mathbf{x}} + \Re\{\mathbf{P}_y\}\hat{\mathbf{y}}$ are shown, and by Fig. (6) which shows the space distribution of the quantity $\Delta W = 1/4(\mu_0|\mathbf{H}|^2 - \epsilon_0|\mathbf{E}|^2)$. We see that the difference between magnetic and electric energy

densities is far from being identically null. This confirms that the field (8) is a *partially standing* wave. Once again, exactly like in the other cases discussed in the previous Sections, the crucial difference between a standing wave and a through-flowing beam is that the equal-amplitude loci of the first one must not be confused with the surfaces orthogonal to the Poynting vector of the second one.

6 Conclusion.

We tried to shed new light on an old problem, namely, whether the idea of a guided mode traveling without any loss through a dielectric taper can be sustained without running into any physical paradox. Our numerical results, obtained with an extended BPM technique, have fully reconfirmed what was stated in ref.[2]: in Marcatili's tapers, standing waves have the basic features outlined in ref.[1], but through-flowing waves do not. This prevents traveling waves from running into a paradox, but on the other hand entails some loss radiation. We have provided an explanation for the unexpected and puzzling result, the drastic difference between standing and through-flowing waves in the same structures. The source of these "surprise" is built into Maxwell's equations.

It was pointed out in ref.[2] that some of the problems discussed here with reference to Marcatili's tapers apply to Gaussian beams in free space as well. Indeed, in the rest of this paper we have discussed Gaussian beams and free-space modes expressed in terms of Bessel functions, and reached essentially the same conclusions as for Marcatili's tapers.

Acknowledgment

We gratefully acknowledge the contribution given to the subject of Section 2 by Mr. Stefano Corrias, who passed away in August 24, 1997.

Figure captions

Fig.(1). Constant-amplitude plots of two standing waves in a superlinear Marcatili's taper, of even (part a)) and odd (part b)) symmetry, with respect to the waist plane.

Fig.(2). Phase fronts (part a)), and field-amplitude contour plot (part b)) for a through-flowing wave in the same superlinear taper as in Fig.(1).

Fig.(3). Power vs. distance, in a superlinear taper of the shape shown in the previous figures whose paramemters are specified in the text.

Fig.(4). Circular cylinder coordinate system.

Fig.(5). Flow lines for the real part of the Poynting vector of free space modes. In inset a) the Poynting vector has been computed as $\mathbf{P} = |\mathbf{E}|^2/2\eta_0$. Whereas, in inset b) it has been computed as $\mathbf{P} = \mathbf{E} \times \mathbf{H}^*/2$.

Fig.(6). Space distribution of the difference ΔW between magnetic and electric energy densities.

References

[1] E.A.J. Marcatili, "Dielectric tapers with curved axes and no loss", *IEEE J. Quant. Electron.*, vol. 21, pp. 307-314, Apr. 1985.

[2] E.A.J. Marcatili and C.G. Someda, "Gaussian beams are fundamentally different from free-space modes", *IEEE J. Quant. Electron.*, vol. 231, pp. 164-167, Feb. 1987.

[3] O. Mtomi, K. Kasaya and H. Miyazawa, "Design of a single-mode tapered waveguide for low-loss chip-to-fiber coupling", *IEEE J. Quant. Electron.*, vol. 30, pp. 1787-1793, Aug. 1994.

[4] I. Mansour and C.G. Someda, "Numerical optimization procedure for low-loss sharp bends in MgO co-doped $Ti - LiNbO_3$ waveguides", *IEEE Photon. Technol. Lett.*, vol. 7, pp. 81-83, Jan. 1995.

[5] C. Vassallo, "Analysis of tapered mode transformers for semiconductor optical amplifiers", *Optical and Quantum Electron.*, vol. 26, 1994.

[6] M.L. Wu, P.L. Fan, J.M. Hsu and C.T. Lee, "Design of ideal structures for lossless bends in optical waveguides by conformal mapping", *IEEE J. Lightwave Technol.*, vol. 14, pp. 2604-2614, Nov. 1996.

[7] C.T. Lee, M.L. Wu, L.G. Sheu, P.L. Fan and J.M. Hsu, "Design and analysis of completely adiabatic tapered waveguides by conformal mapping", *IEEE J. Lightwave Technol.*, vol. 15, pp. 403-410, Feb. 1997.

[8] J.I. Sakai and E.A.J. Marcatili, “Lossless dielectric tapers with three-dimensional geometry”, *IEEE J. Lightwave Technol.*, vol. 9, pp. 386-393, Mar. 1991.

[9] A. Yariv, *Introduction to Optical Electronics*, 2nd Edition. Holt, Rinehart and Winston, New York, 1976.

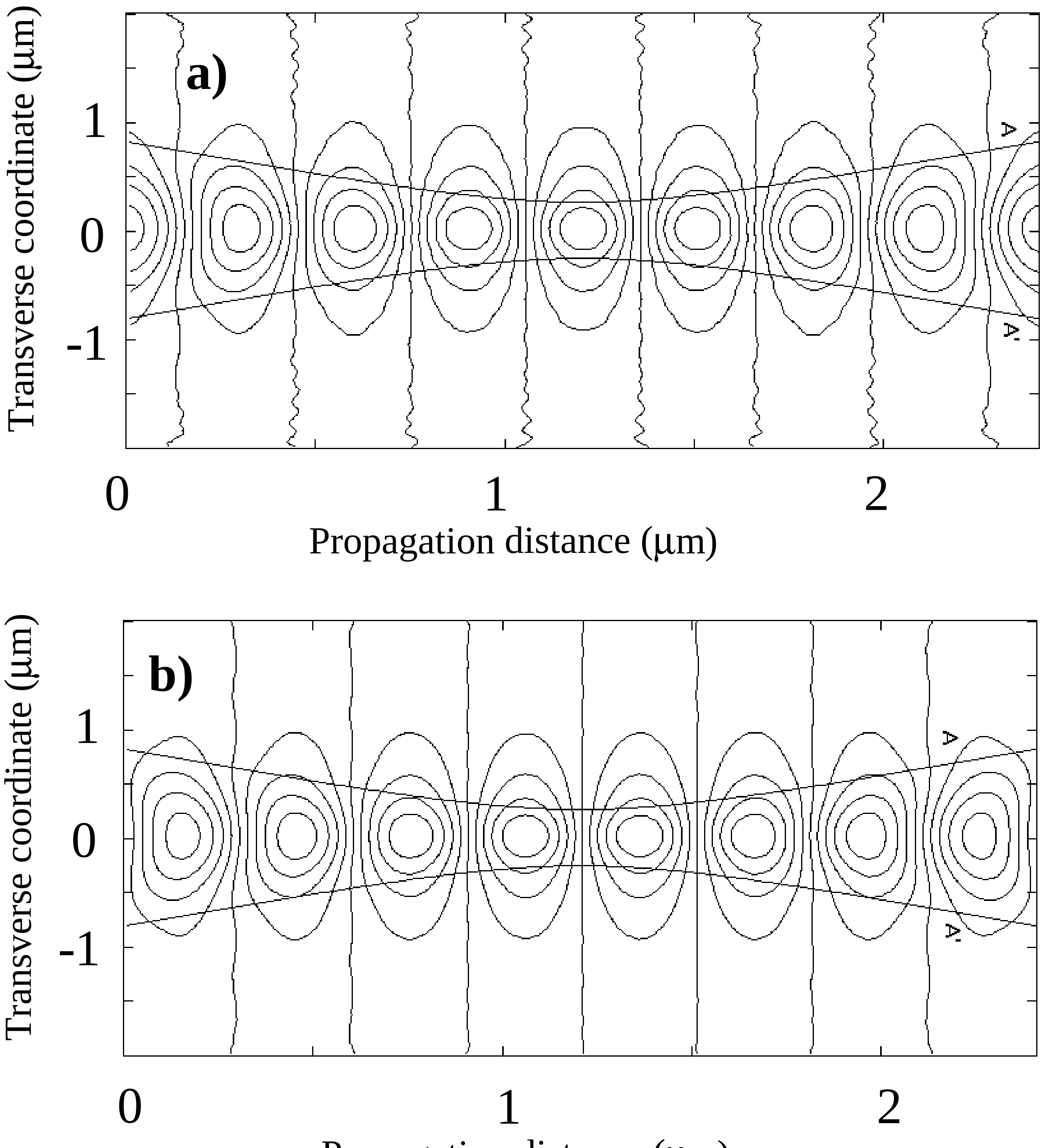
a)
b)
Transverse coordinate (μm)
1
0
-1
A
A'
0
1
2
Propagation distance (μm)

Figure (1)

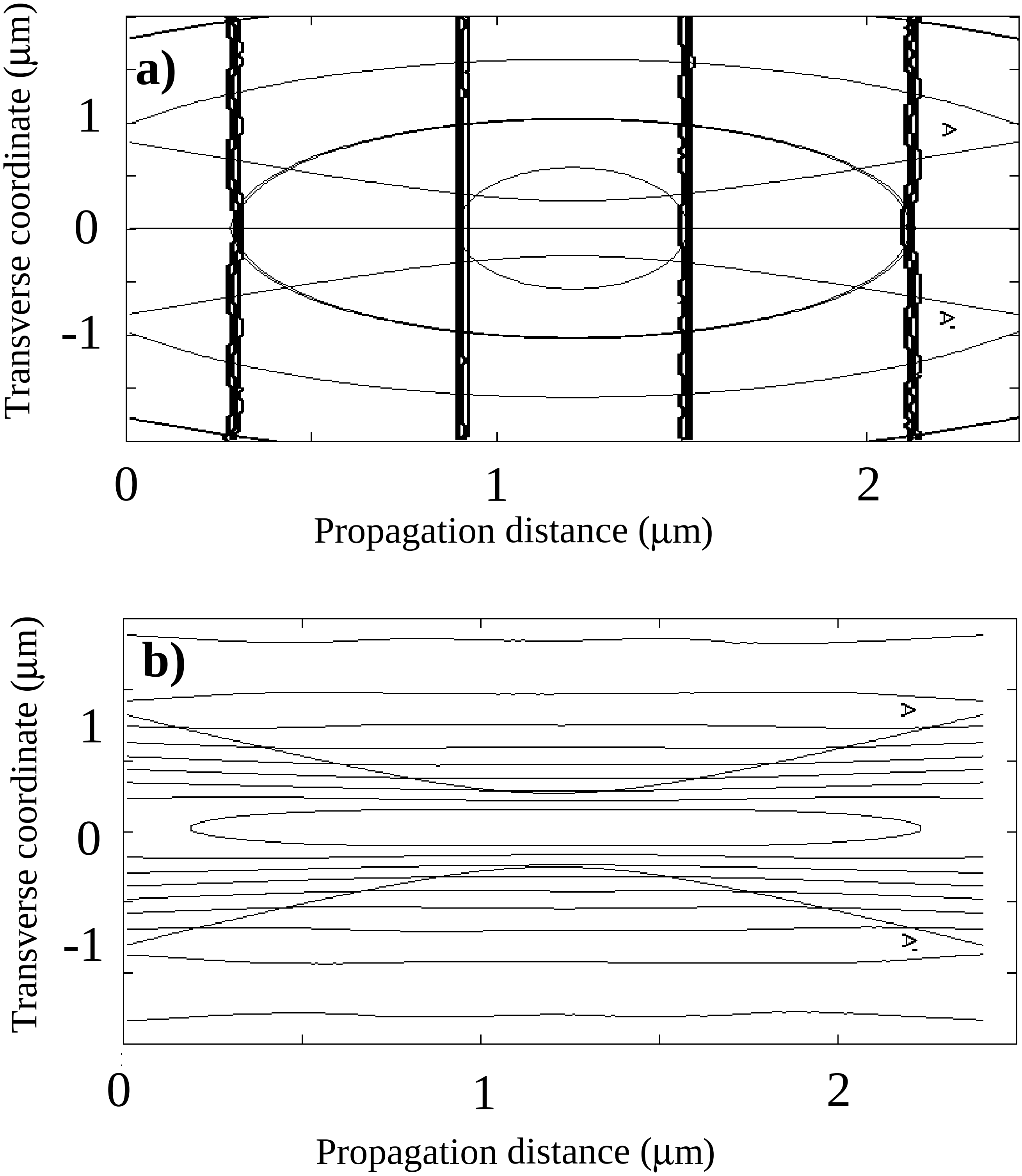
a)
Transverse coordinate (μm)
1
0
-1
A
A'
0
1
2
Propagation distance (μm)
b)
Transverse coordinate (μm)
1
0
-1
A
A'
0
1
2
Propagation distance (μm)

Figure (2)

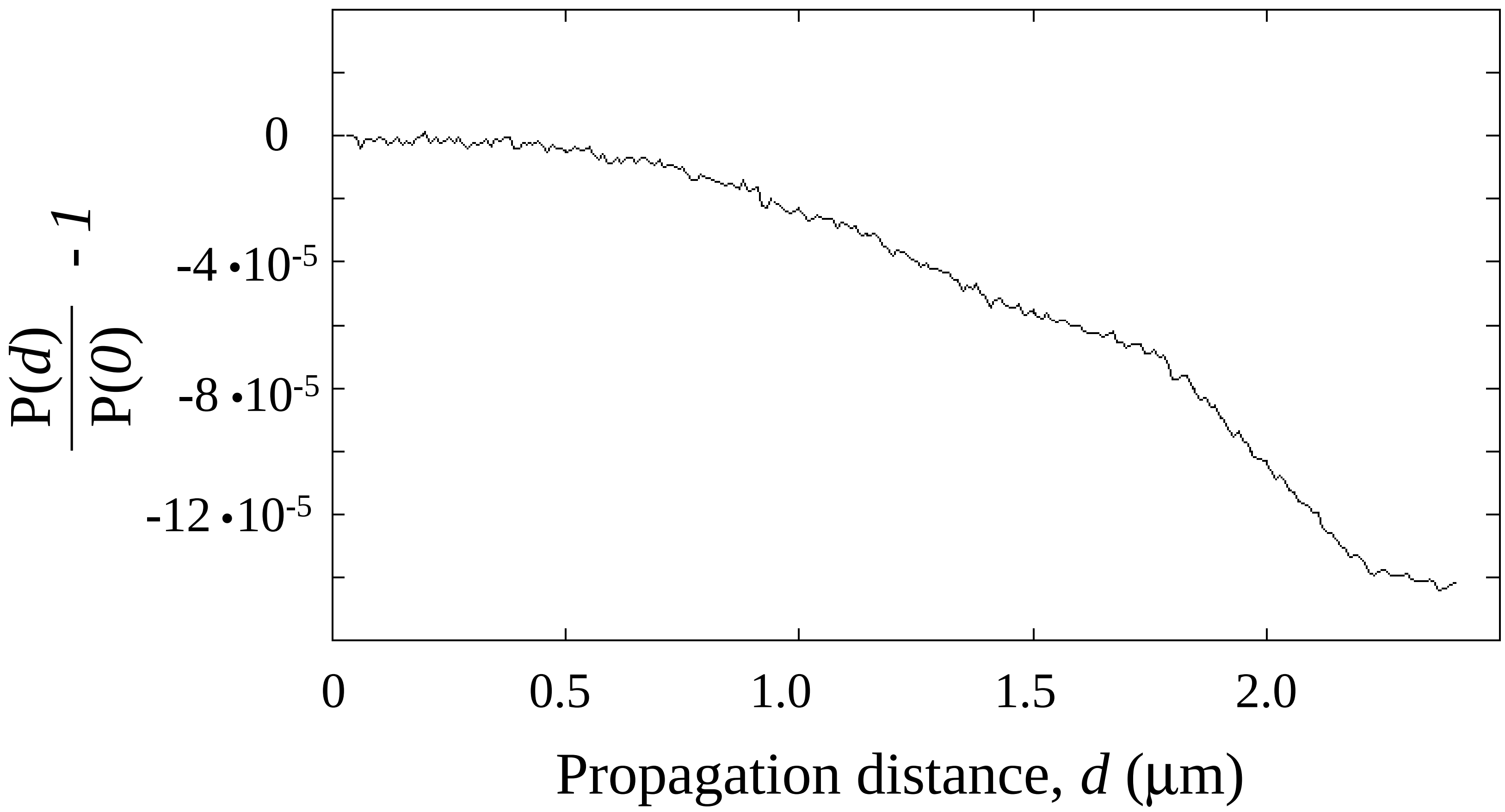
0
-4 •10-5
-8 •10-5
-12 •10-5
P(d) / P(0) - 1
0
0.5
1.0
1.5
2.0
Propagation distance, d (μm)

Figure (3)

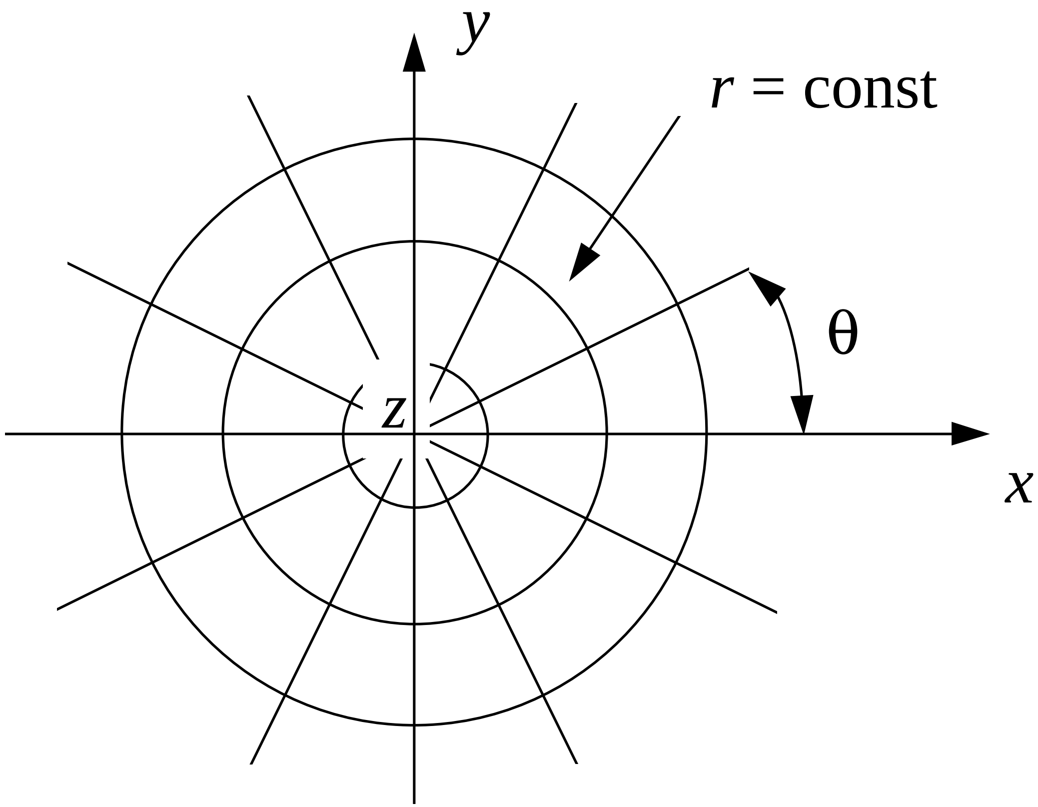

Figure (4)

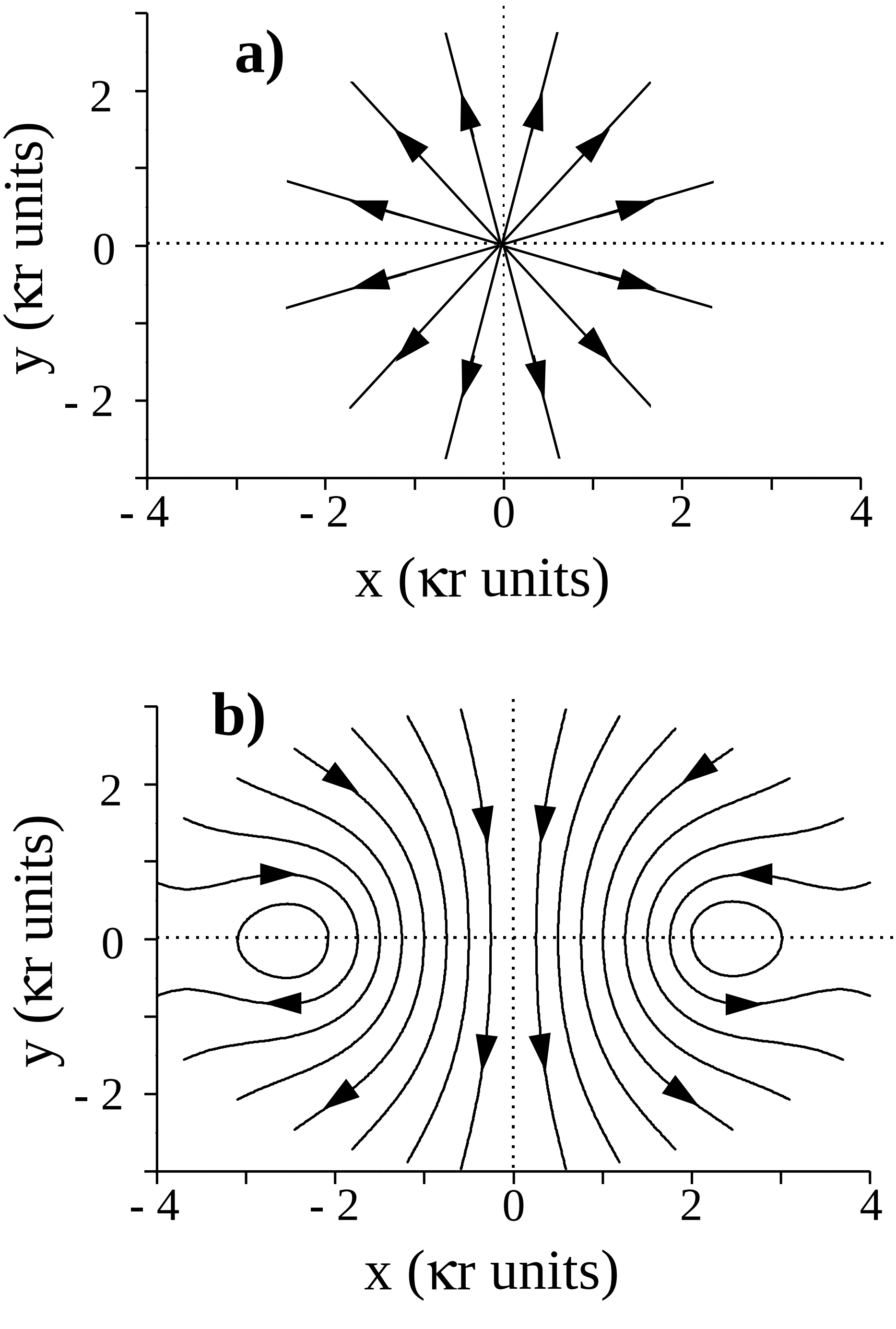
a)
2
0
- 2
y (κr units)
- 4
- 2
0
2
4
x (κr units)
b)
2
0
- 2
y (κr units)
- 4
- 2
0
2
4
x (κr units)

Figure (5)

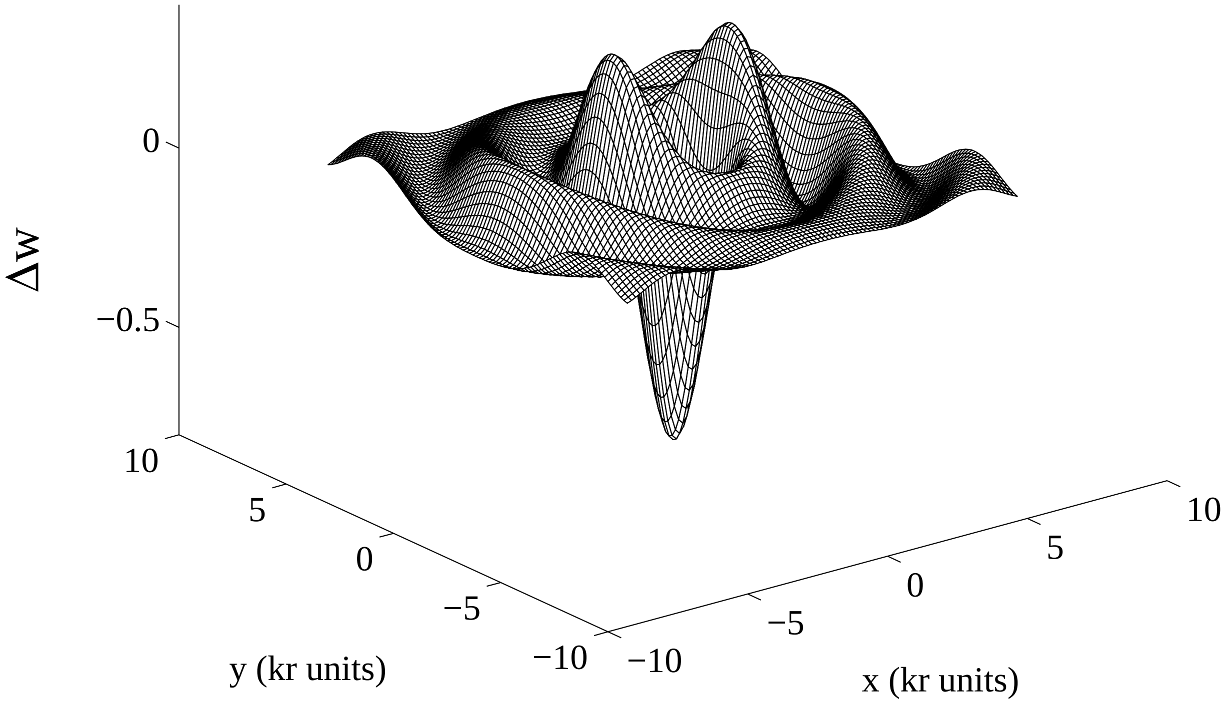

Figure (6)